Quadratic Equations

1. Determine the nature of the roots:

$a.\ 4x^2 - 5x + 7 = b^2 - 4ac = (-5)^2 - 4(4)(7) = -31 \Rightarrow 2$ imaginary roots

$b.\ -3x^2 + 2x + 9 = b^2 - 4ac = (2)^2 - 4(-3)(9) = 112 \Rightarrow 2$ real roots

$c.\ 5x^2 + 2x - 7 = b^2 - 4ac = (2)^2 - 4(5)(-7) = 144 \Rightarrow 2$ real roots

2. Determine the sum and product of the roots of the following:

$a.\ 6x^2 - 3x - 7 \Rightarrow sum = -\dfrac{b}{a} = -\dfrac{(-3)}{6} = \dfrac{3}{6} = \dfrac{1}{2}, product = \dfrac{c}{a} = \dfrac{(-7)}{6} = -\dfrac{7}{6}$

$b.\ -4x^2 + 5x - 1 \Rightarrow sum = -\dfrac{b}{a} = -\dfrac{(5)}{(-4)} = \dfrac{5}{4}, product = \dfrac{c}{a} = \dfrac{(-1)}{(-4)} = \dfrac{1}{4}$

$c.\ 2x^2 - 6x - 7 \Rightarrow sum = -\dfrac{b}{a} = -\dfrac{(-6)}{2} = \dfrac{6}{2} = 3, product = \dfrac{c}{a} = \dfrac{(-7)}{2} = -\dfrac{7}{2}$

3. Determine the roots of the following quadratic equations using factoring:

$a.\ x^2 - 7x - 18 = 0 \Rightarrow (x-9)(x+2) = 0 \Rightarrow x = 9$ or $-2 \Rightarrow \{-2,9\}$

$b.\ x^2 + 3x - 10 = 0 \Rightarrow (x-2)(x+5) = 0 \Rightarrow x = 2$ or $-5 \Rightarrow \{-5,2\}$

$c.\ 2x^2 + 15x + 7 = 0 \Rightarrow (2x+1)(x+7) = 0 \Rightarrow x = -\dfrac{1}{2}$ or $-7 \Rightarrow \left\{-7,-\dfrac{1}{2}\right\}$

$d.\ -3x^2 + 2x + 5 = 0 \Rightarrow 3x^2 - 2x - 5 = 0 \Rightarrow (3x-5)(x+1) = 0 \Rightarrow x = \dfrac{5}{3}$ or $-1 \Rightarrow \left\{-1,\dfrac{5}{3}\right\}$

4. Determine the roots of the following quadratic equations using the quadratic formula:

$a.\ x^2 - 5x - 36 = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-36)}}{2(1)} = \dfrac{5 \pm \sqrt{25 + 144}}{2} =$

$\dfrac{5 \pm \sqrt{169}}{2} = \dfrac{5 \pm 13}{2} = \dfrac{5 + 13}{2}$ or $\dfrac{5 - 13}{2} = 9$ or $-4 \Rightarrow \{-4,9\}$

$b.\ 3x^2 - 4x + 11 = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(11)}}{2(3)} = \dfrac{4 \pm \sqrt{16 - 132}}{6} =$

$\dfrac{4 \pm \sqrt{-116}}{6} = \dfrac{4 \pm 2i\sqrt{29}}{6} \Rightarrow \left\{\dfrac{4 \pm 2i\sqrt{29}}{6}\right\}$

$c.\ -2x^2 + 3x + 5 = 0\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(-2)(5)}}{2(-2)} = \dfrac{-3 \pm \sqrt{9 + 40}}{-4} =$

$\dfrac{-3 \pm \sqrt{49}}{-4} = \dfrac{5 \pm 7}{-4} = \dfrac{5 + 7}{-4}$ or $\dfrac{5 - 7}{-4} = -3$ or $\dfrac{1}{2} \Rightarrow \left\{-3,\dfrac{1}{2}\right\}$

$d.\ 5x^2 + 12x - 4 = 0 \Rightarrow x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \dfrac{-(-12) \pm \sqrt{(12)^2 - 4(5)(-4)}}{2(5)} =$

$\dfrac{12 \pm \sqrt{144 + 80}}{10} = \dfrac{12 \pm \sqrt{224}}{10} = \dfrac{5 \pm 4\sqrt{14}}{2} \Rightarrow \left\{ \dfrac{5 \pm 4\sqrt{14}}{2} \right\}$

5. Determine the equation given the following roots (use the concept of factors):

$a.\ \{3,6\} \Rightarrow (x - 3)(x - 6) = 0 \Rightarrow x^2 - 9x + 18 = 0$

$b.\ \{-6,-5\} \Rightarrow (x + 6)(x + 5) = 0 \Rightarrow x^2 + 11x + 30 = 0$

$c.\ \{-4i, 4i\} \Rightarrow (x + 4i)(x - 4i) = 0 \Rightarrow x^2 - 4ix + 4ix - 16i^2 = 0 \Rightarrow x^2 + 16 = 0$

$d.\ \{5 - 3i, 5 + 3i\} \Rightarrow [x - (5 - 3i)][x - (5 + 3i)] = 0 \Rightarrow x^2 - x(5 + 3i) - x(5 - 3i) + (5 - 3i)(5 + 3i) = 0 \Rightarrow$

$\quad x^2 - x(5 + 3i) - x(5 - 3i) + (5 - 3i)(5 + 3i) = 0 \Rightarrow x^2 - 5x - 3ix - 5x + 3ix + 25 - 9i^2 = 0 \Rightarrow$

$\quad x^2 - 10x + 34 = 0$

$e.\ \left\{ \dfrac{2i - 3}{4}, \dfrac{2i + 3}{4} \right\} \Rightarrow \left[x - \left(\dfrac{2i - 3}{4} \right) \right]\left[x - \left(\dfrac{2i + 3}{4} \right) \right] = 0 \Rightarrow$

$\quad x^2 - x\left(\dfrac{2i - 3}{4} \right) - x\left(\dfrac{2i + 3}{4} \right) + \left(\dfrac{2i - 3}{4} \right)\left(\dfrac{2i + 3}{4} \right) = 0 \Rightarrow x^2 - x(5 + 3i) - x(5 - 3i) + (5 - 3i)(5 + 3i) = 0 =$

$\quad x^2 - \dfrac{2}{4}ix + \dfrac{3}{4}x - \dfrac{2}{4}ix - \dfrac{3}{4}x + \dfrac{4i^2 - 9}{16} = 0 \Rightarrow x^2 - 10x + 34 = 0$

6. Determine the equation given the following roots (use the sum and product formula):

$a.\ \{-2,6\} \Rightarrow x^2 - (r_1 + r_2)x + r_1 r_2 = 0 \Rightarrow x^2 - (-2 + 6)x + (-2)(6) = 0 \Rightarrow x^2 - 4x - 12 = 0$

$b.\ \{-4,-7\} \Rightarrow x^2 - (r_1 + r_2)x + r_1 r_2 = 0 \Rightarrow x^2 - [-4 + (-7)]x + (-4)(-7) = 0 \Rightarrow x^2 + 11x + 28 = 0$

$c.\ \{-5i, 5i\} \Rightarrow x^2 - (r_1 + r_2)x + r_1 r_2 = 0 \Rightarrow x^2 - (-5i + 5i)x + (-5i)(5i) = 0 \Rightarrow x^2 - 25i^2 = 0 \Rightarrow x^2 + 25 =$

$d.\ \{2 - 5i, 2 + 5i\} \Rightarrow x^2 - (r_1 + r_2)x + r_1 r_2 = 0 \Rightarrow x^2 - [(2 - 5i) + (2 + 5i)]x + (2 - 5i)(2 + 5i) = 0 \Rightarrow$

$x^2 - 4x + 4 - 25i^2 = 0 \Rightarrow x^2 - 4x + 29 = 0$

$e.\ \left\{ \dfrac{7i - 4}{2}, \dfrac{7i + 4}{2} \right\} \Rightarrow x^2 - (r_1 + r_2)x + r_1 r_2 = 0 \Rightarrow x^2 - \left(\dfrac{7i - 4}{2} + \dfrac{7i + 4}{2} \right)x + \left(\dfrac{7i - 4}{2} \right)\left(\dfrac{7i + 4}{2} \right) = 0 \Rightarrow$

$x^2 - \left(\dfrac{14i}{2} \right)x - \left(\dfrac{49i^2 - 16}{4} \right) = 0 \Rightarrow x^2 - \dfrac{14i}{2}x + \dfrac{65}{4} = 0 \Rightarrow 4x^2 - 28i + 65 = 0$

$f.\ \left\{ \dfrac{3 \pm 2i}{5} \right\} x^2 - (r_1 + r_2)x + r_1 r_2 = 0 \Rightarrow x^2 - \left(\dfrac{3 - 2i}{5} + \dfrac{3 + 2i}{5} \right)x + \left(\dfrac{3 - 2i}{5} \right)\left(\dfrac{3 + 2i}{5} \right) = 0 \Rightarrow$

$x^2 - \left(\dfrac{6}{5} \right)x - \left(\dfrac{9 - 4i^2}{25} \right) = 0 \Rightarrow x^2 - \dfrac{6}{5}x - \dfrac{13}{25} = 0 \Rightarrow 25x^2 - 30x - 13 = 0$